Chapter One

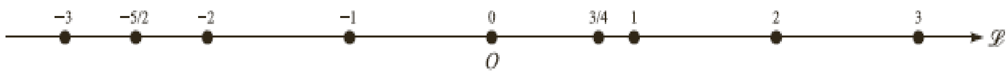
Basic Principles and Review

1.1 ***COORDINATE SYSTEMS*** :-

a) ***Coordinate System on a Line:***

Let L be a line. Choose a point O on the line and call this point the origin and select a direction along L; say, the direction from left to right on the diagram. For every point P to the right of the origin O, let the coordinate of P be the distance between O and P.

In the diagram: The origin O is assigned the number (0) as its coordinate. This assignment of real numbers to the points on the line L is called a coordinate system on L.



Choosing a different origin, a different direction along the line, or a different unit distance would result in a different coordinate system.

b) ***Cartesian Coordinate in a Plane:***

By found a correspondence between the points of a plane and pairs of real numbers. Choose two perpendicular lines in the plane of Fig. 2-1. Let us assume for the sake of simplicity that one of the lines is horizontal and the other vertical. The horizontal line will be called the *x*-axis and the vertical line will be called the *y*-axis.

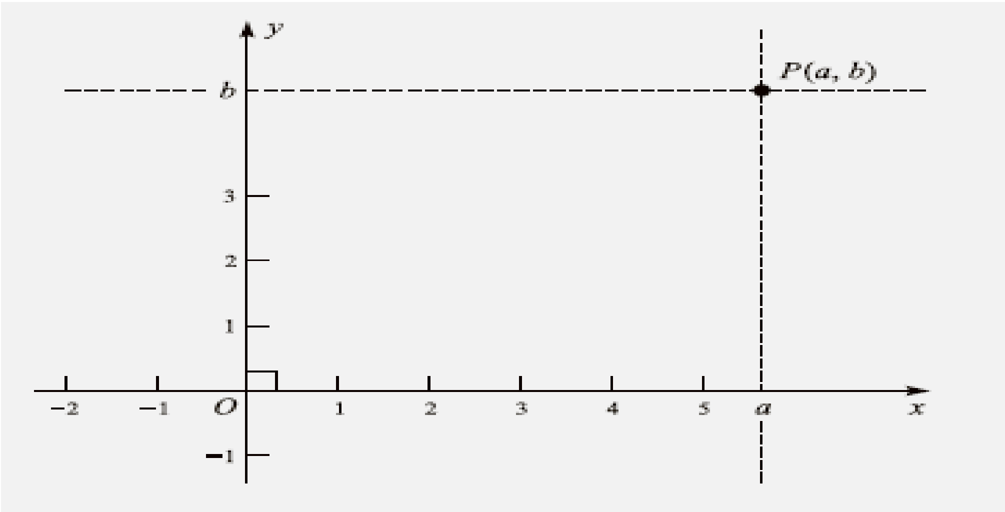


Fig. 2-1

Next choose a coordinate system on the *x*-axis and one on the *y*-axis. The origin for both coordinate systems is taken to be the point *O*, where the axes intersect. The *x*-axis is directed from left to right, the *y*-axis from bottom to top. Consider any point *P* in the plane. Take the vertical line through the point *P*, and let be the coordinate of the point where the line intersects the *x*-axis. The number *b* is called the *y coordinate* of *P*. Every point has a unique pair (*a*, *b*) of coordinates associated with it.

EXAMPLES In Fig. 2-2, the coordinates of several points have been indicated. We have limited ourselves to integer coordinates only for simplicity.

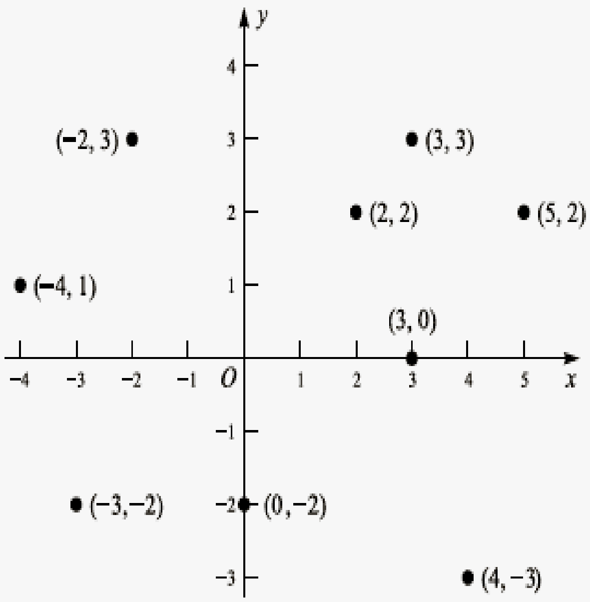


Fig. 2-2

Conversely, every pair (*a*, *b*) of real numbers is associated with a unique point in the plane.

Absolute Value

The **absolute value** of a number x, denoted by $|x|$, is defined by the formula

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0. \end{cases}$$

EXAMPLE Finding Absolute Values

$$|3| = 3, \quad |0| = 0, \quad |-5| = -(-5) = 5, \quad |-|a|| = |a|$$

Geometrically, the absolute value of x is the distance from x to 0 on the real number line. Since distances are always positive or 0, we see that $|x| \geq 0$ for every real number x, and $|x| = 0$ if and only if $x = 0$. Also,

$$|x - y| = \text{the distance between } x \text{ and } y$$

on the real line

Since the symbol $\sqrt{a}$ always denotes the *nonnegative* square root of a, an alternate definition of $|x|$ is

$$|x| = \sqrt{x^2}.$$

It is important to remember that $\sqrt{a^2} = |a|$. Do not write $\sqrt{a^2} = a$ unless you already know that $a \geq 0$.

Absolute Value Properties

1. $|-a| = |a|$	A number and its additive inverse or negative have the same absolute value.
2. $|ab| = |a||b|$	The absolute value of a product is the product of the absolute values.
3. $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}$	The absolute value of a quotient is the quotient of the absolute values.
4. $|a + b| \leq |a| + |b|$	The **triangle inequality**. The absolute value of the sum of two numbers is less than or equal to the sum of their absolute values.

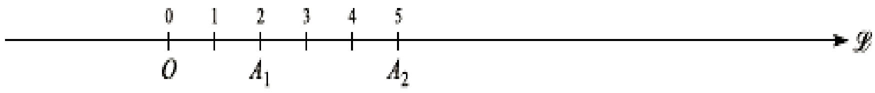
Absolute Value and Distance

Consider a coordinate system on a line $\mathscr{L}$ and let A_1 and A_2 be points on $\mathscr{L}$ with coordinates a_1 and a_2. Then

$$|a_1 - a_2| = \overline{A_1A_2} = \text{distance between } A_1 \text{ and } A_2 \tag{1.6}$$

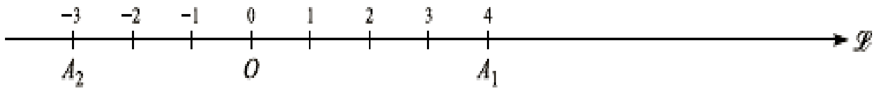
EXAMPLES

(a)



$$|a_1 - a_2| = |2 - 5| = |-3| = 3 = \overline{A_1A_2}$$

(b)



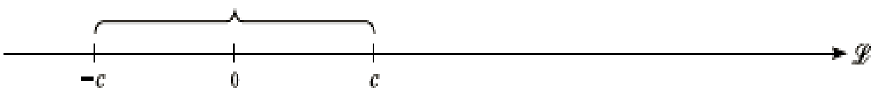
$$|a_1 - a_2| = |4 - (-3)| = |4 + 3| = |7| = 7 = \overline{A_1A_2}$$

A special case of (1.6) is very important. If a is the coordinate of A, then

$$|a| = \text{distance between } A \text{ and the origin} \tag{1.7}$$

Notice that, for any positive number c,

$$|u| \le c \quad \text{is equivalent to} \quad -c \le u \le c \tag{1.8}$$



EXAMPLE Illustrating the Triangle Inequality

$$|-3 + 5| = |2| = 2 < |-3| + |5| = 8$$
$$|3 + 5| = |8| = |3| + |5|$$
$$|-3 - 5| = |-8| = 8 = |-3| + |-5|$$

The inequality $|x| < a$ says that the distance from x to 0 is less than the positive number a. This means that x must lie between $-a$ and a,

EXAMPLE Solving an Equation with Absolute Values

Solve the equation $|2x - 3| = 7$.

Solution By Property 5, $2x - 3 = \pm 7$, so there are two possibilities:

$$\begin{array}{rclcrcll} 2x - 3 &=& 7 & \quad & 2x - 3 &=& -7 & \quad \text{Equivalent equations without absolute values} \\ 2x &=& 10 & & 2x &=& -4 & \quad \text{Solve as usual.} \\ x &=& 5 & & x &=& -2 & \end{array}$$

The solutions of $|2x - 3| = 7$ are $x = 5$ and $x = -2$.

EXAMPLE Solving an Inequality Involving Absolute Values

Solve the inequality $\left|5 - \frac{2}{x}\right| < 1$.

Solution We have

$$\begin{aligned} \left|5 - \frac{2}{x}\right| < 1 &\Leftrightarrow -1 < 5 - \frac{2}{x} < 1 && \text{Property 6} \\ &\Leftrightarrow -6 < -\frac{2}{x} < -4 && \text{Subtract 5.} \\ &\Leftrightarrow 3 > \frac{1}{x} > 2 && \text{Multiply by } -\tfrac{1}{2}. \\ &\Leftrightarrow \frac{1}{3} < x < \frac{1}{2}. && \text{Take reciprocals.} \end{aligned}$$

EXAMPLE Solve the inequality and show the solution set on the real line:

(a) $|2x - 3| \le 1$ **(b)** $|2x - 3| \ge 1$

Solution

(a)

$$\begin{aligned} |2x - 3| &\le 1 \\ -1 \le 2x - 3 &\le 1 && \text{Property 8} \\ 2 \le 2x &\le 4 && \text{Add 3.} \\ 1 \le x &\le 2 && \text{Divide by 2.} \end{aligned}$$

The solution set is the closed interval $[1, 2]$ (Figure 1.4a).

(b)

$$|2x - 3| \ge 1$$

$$\begin{array}{rcrl} 2x - 3 \ge 1 & \text{or} & 2x - 3 \le -1 & \quad \text{Property 9} \\ x - \frac{3}{2} \ge \frac{1}{2} & \text{or} & x - \frac{3}{2} \le -\frac{1}{2} & \quad \text{Divide by 2.} \\ x \ge 2 & \text{or} & x \le 1 & \quad \text{Add } \frac{3}{2}. \end{array}$$

2.2 ***Increments and Straight Lines***:

When a particle moves from one point in the plane to another, the net changes in its coordinates are called *increments.* They are calculated by subtracting the coordinates of the starting point from the coordinates of the ending point. If *x* changes from to the increment in *x* is :

$$\Delta \mathbf{x} = \mathbf{x_2} - \mathbf{x_1}$$

EXAMPLE 1 In going from the point $A(4, -3)$ to the point $B(2, 5)$ the increments in the x- and y-coordinates are

$$\Delta x = 2 - 4 = -2, \qquad \Delta y = 5 - (-3) = 8.$$

From $C(5, 6)$ to $D(5, 1)$ the coordinate increments are

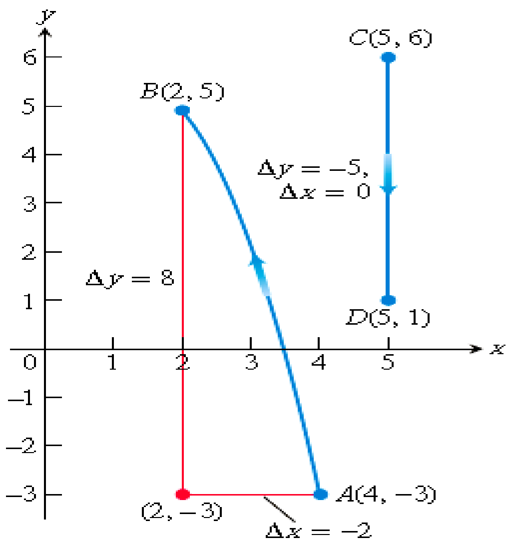
$$\Delta x = 5 - 5 = 0, \qquad \Delta y = 1 - 6 = -5.$$

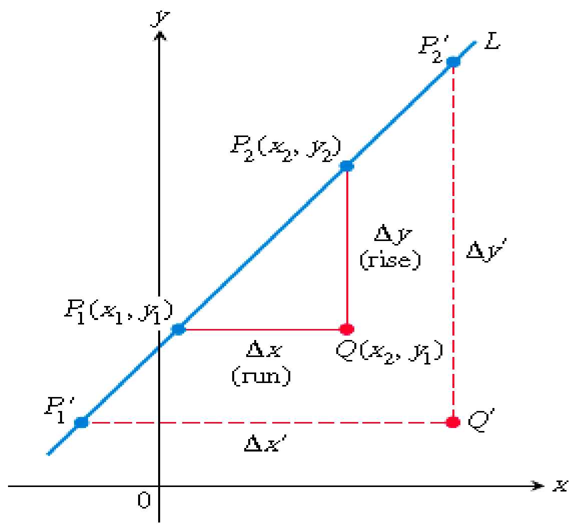
Given two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in the plane, we call the increments $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$ the **run** and the **rise**, respectively, between P_1 and P_2. Two such points always determine a unique straight line (usually called simply a line) passing through them both. We call the line P_1P_2.

Any nonvertical line in the plane has the property that the ratio

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

has the same value for every choice of the two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ on the line This is because the ratios of corresponding sides for similar triangles are equal.





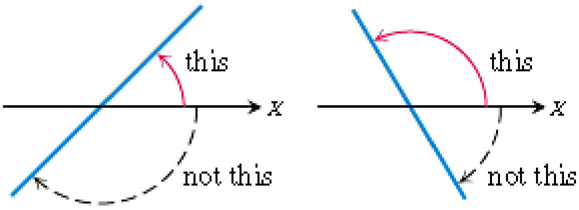
DEFINITION Slope

The constant

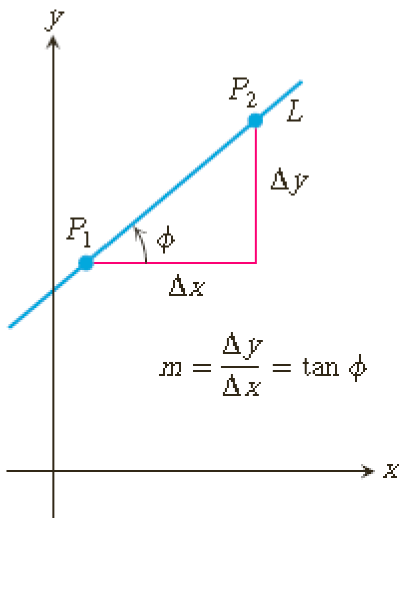
$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

is the **slope** of the nonvertical line P_1P_2.

The slope tells us the direction of a line. The direction of a line can be measured with an angle. The angle of a line that crosses the *x*-axis is the smallest counterclockwise angle from the *x*-axis to the line as shown in figure.



The relationship between the slope *m* of a non vertical line and the line's angle of inclination ∅ is shown in Figure:



We can write an equation for a non vertical straight line L, if we know its slope m and the coordinates of one point $p_1(x1, y1)$on it . If p (x, y) is any other point on L, then we can use the two points p_1 and p to compute the slope,

$$m = \frac{y - y_1}{x - x_1}$$

so that

$$y - y_1 = m(x - x_1) \qquad \text{or} \qquad y = y_1 + m(x - x_1).$$

> The equation
>
> $$y = y_1 + m(x - x_1)$$
>
> is the **point-slope equation** of the line that passes through the point (x_1, y_1) and has slope m.

EXAMPLE 2 Write an equation for the line through the point $(2, 3)$ with slope $-3/2$.

Solution We substitute $x_1 = 2, y_1 = 3$, and $m = -3/2$ into the point-slope equation and obtain

$$y = 3 - \frac{3}{2}(x - 2), \qquad \text{or} \qquad y = -\frac{3}{2}x + 6.$$

When $x = 0$, $y = 6$ so the line intersects the y-axis at $y = 6$. ■

EXAMPLE 3 A Line Through Two Points

Write an equation for the line through $(-2, -1)$ and $(3, 4)$.

Solution The line's slope is

$$m = \frac{-1 - 4}{-2 - 3} = \frac{-5}{-5} = 1.$$

We can use this slope with either of the two given points in the point-slope equation:

With $(x_1, y_1) = (-2, -1)$	With $(x_1, y_1) = (3, 4)$
$y = -1 + 1 \cdot (x - (-2))$	$y = 4 + 1 \cdot (x - 3)$
$y = -1 + x + 2$	$y = 4 + x - 3$
$y = x + 1$	$y = x + 1$

Same result

Either way, $y = x + 1$ is an equation for the line

The equation

$$y = mx + b$$

is called the **slope-intercept equation** of the line with slope m and y-intercept b.

Lines with equations of the form $y = mx$ have y-intercept 0 and so pass through the origin. Equations of lines are called **linear** equations.

The equation

$$Ax + By = C \qquad (A \text{ and } B \text{ not both } 0)$$

is called the **general linear equation** in x and y because its graph always represents a line and every line has an equation in this form (including lines with undefined slope).

EXAMPLE 4 Finding the Slope and y-Intercept

Find the slope and y-intercept of the line $8x + 5y = 20$.

Solution Solve the equation for y to put it in slope-intercept form:

$$\begin{aligned} 8x + 5y &= 20 \\ 5y &= -8x + 20 \\ y &= -\frac{8}{5}x + 4. \end{aligned}$$

The slope is $m = -8/5$. The y-intercept is $b = 4$.

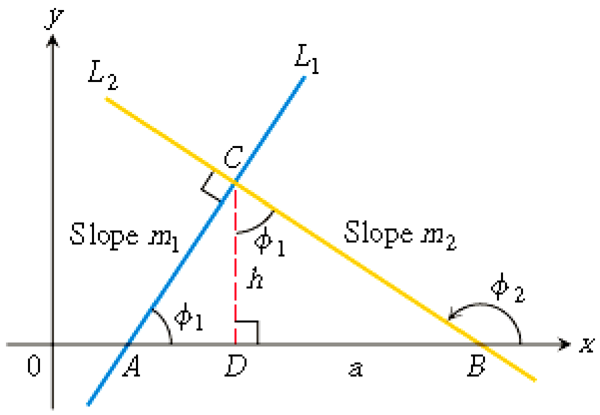
Parallel and Perpendicular Lines

Lines that are parallel have equal angles of inclination, so they have the same slope (if they are not vertical). Conversely, lines with equal slopes have equal angles of inclination and so are parallel.

If two nonvertical lines L_1 and L_2 are perpendicular, their slopes m_1 and m_2 satisfy $m_1 m_2 = -1$, so each slope is the *negative reciprocal* of the other:

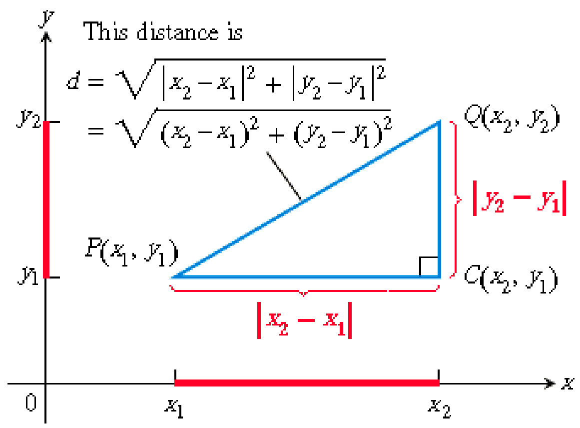
$$m_1 = -\frac{1}{m_2}, \qquad m_2 = -\frac{1}{m_1}.$$

To see this, notice by inspecting similar triangles in Figure that $m_1 = a/h$, and $m_2 = -h/a$. Hence, $m_1 m_2 = (a/h)(-h/a) = -1$.



Distance and Circles in the Plane

The distance between points in the plane is calculated with a formula that comes from the Pythagorean theorem



EXAMPLE 5 Calculating Distance

(a) The distance between $P(-1, 2)$ and $Q(3, 4)$ is

$$\sqrt{(3 - (-1))^2 + (4 - 2)^2} = \sqrt{(4)^2 + (2)^2} = \sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}.$$

(b) The distance from the origin to $P(x, y)$ is

$$\sqrt{(x - 0)^2 + (y - 0)^2} = \sqrt{x^2 + y^2}.$$

By definition, a **circle** of radius a is the set of all points $P(x, y)$ whose distance from some center $C(h, k)$ equals a From the distance formula, P lies on the circle if and only if

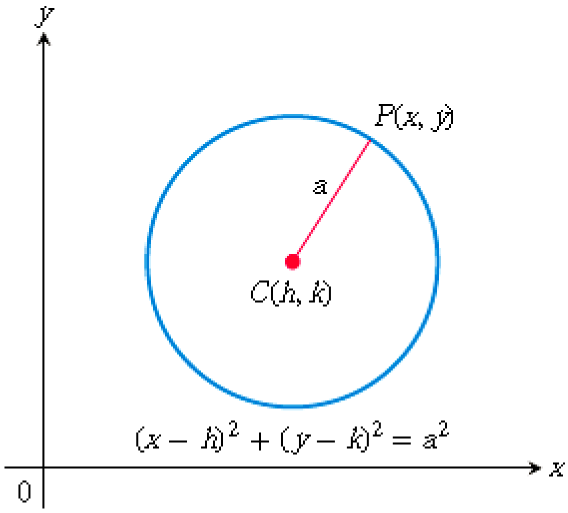
$$\sqrt{(x - h)^2 + (y - k)^2} = a,$$

so

$$(x - h)^2 + (y - k)^2 = a^2. \qquad (1)$$

Equation (1) is the **standard equation** of a circle with center (h, k) and radius a. The circle of radius $a = 1$ and centered at the origin is the **unit circle** with equation

$$x^2 + y^2 = 1.$$



EXAMPLE 6

(a) The standard equation for the circle of radius 2 centered at $(3, 4)$ is

$$(x - 3)^2 + (y - 4)^2 = 2^2 = 4.$$

(b) The circle

$$(x - 1)^2 + (y + 5)^2 = 3$$

has $h = 1, k = -5$, and $a = \sqrt{3}$. The center is the point $(h, k) = (1, -5)$ and the radius is $a = \sqrt{3}$.

If an equation for a circle is not in standard form, we can find the circle's center and radius by first converting the equation to standard form. The algebraic technique for doing so is *completing the square* .

Functions and Their Graphs:

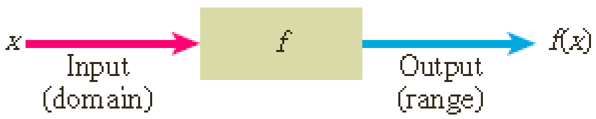
The value of one variable quantity, which we might call (y), depends on the value of another variable quantity, which we might call (x). Since the value of (y), is completely determined by the value of (x), we say that *y* is a function of x. A symbolic way to say "*y* is a function of *x*" is by writing:-

$$y = f(x) \qquad (\text{"}y \text{ equals } f \text{ of } x\text{"})$$

DEFINITION **Function**

A **function** from a set D to a set Y is a rule that assigns a *unique* (single) element $f(x) \in Y$ to each element $x \in D$.

The set *D* of all possible input values is called the domain of the function. The set of all values of $f(x)$ as *x* varies throughout *D* is called the range of the function. The range may not include every element in the set *Y*.



Identifying Domain and Range

Verify the domains and ranges of these functions.

Function	Domain (x)	Range (y)
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = 1/x$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1 - x^2}$	$[-1, 1]$	$[0, 1]$

Solution The formula $y = x^2$ gives a real y-value for any real number x, so the domain is $(-\infty, \infty)$. The range of $y = x^2$ is $[0, \infty)$ because the square of any real number is nonnegative and every nonnegative number y is the square of its own square root, $y = (\sqrt{y})^2$ for $y \geq 0$.

The formula $y = 1/x$ gives a real y-value for every x except $x = 0$. *We cannot divide any number by zero*. The range of $y = 1/x$, the set of reciprocals of all nonzero real numbers, is the set of all nonzero real numbers, since $y = 1/(1/y)$.

The formula $y = \sqrt{x}$ gives a real y-value only if $x \geq 0$. The range of $y = \sqrt{x}$ is $[0, \infty)$ because every nonnegative number is some number's square root (namely, it is the square root of its own square).

In $y = \sqrt{4 - x}$, the quantity $4 - x$ cannot be negative. That is, $4 - x \geq 0$, or $x \leq 4$. The formula gives real y-values for all $x \leq 4$. The range of $\sqrt{4 - x}$ is $[0, \infty)$, the set of all nonnegative numbers.

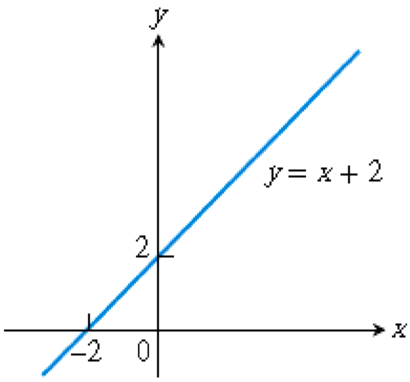
The formula $y = \sqrt{1 - x^2}$ gives a real y-value for every x in the closed interval from -1 to 1. Outside this domain, $1 - x^2$ is negative and its square root is not a real number. The values of $1 - x^2$ vary from 0 to 1 on the given domain, and the square roots of these values do the same. The range of $\sqrt{1 - x^2}$ is [0, 1].

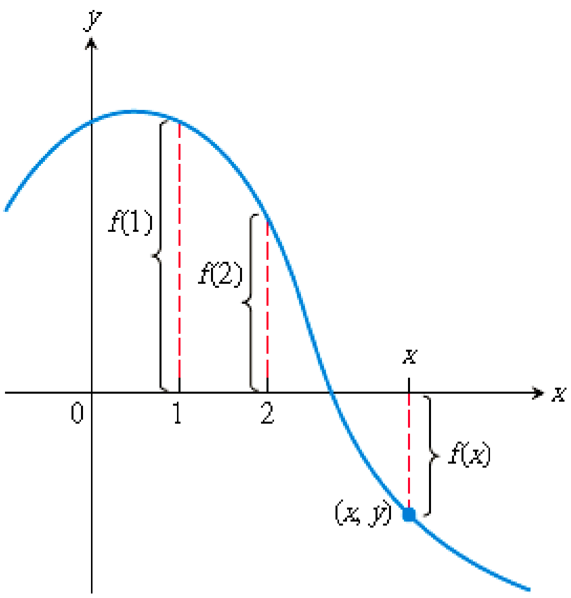
Graphs of Functions:

If f is a function with domain D, its **graph** consists of the points in the Cartesian plane whose coordinates are the input-output pairs for f. In set notation, the graph is

$$\{(x, f(x)) \mid x \in D\}.$$

The graph of a function f is a useful picture of its behavior. If (x, y) is a point on the graph, then $y = f(x)$ is the height of the graph above the point x. The height may be positive or negative, depending on the sign of $f(x)$





EXAMPLE Sketching a Graph

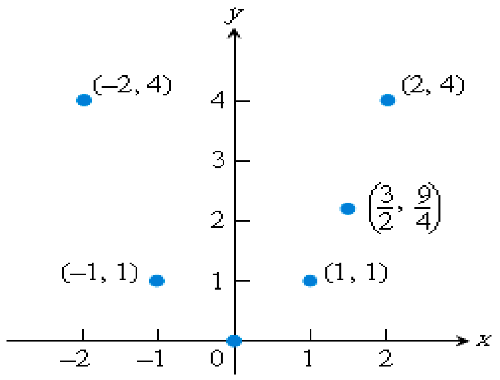
Graph the function $y = x^2$ over the interval $[-2, 2]$.

Solution

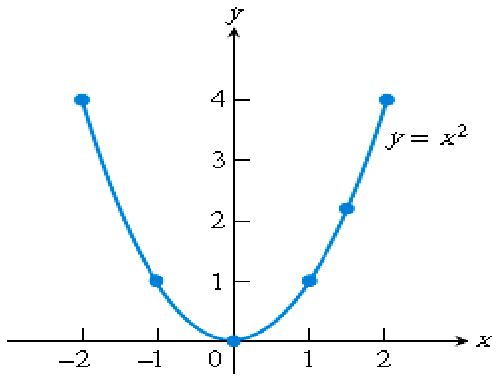
1. Make a table of xy-pairs that satisfy the function rule, in this case the equation $y = x^2$.

x	$y = x^2$
-2	4
-1	1
0	0
1	1
$\frac{3}{2}$	$\frac{9}{4}$
2	4

2. Plot the points (x, y) whose coordinates appear in the table. Use fractions when they are convenient computationally.



3. Draw a smooth curve through the plotted points. Label the curve with its equation.



3- LIMITS AND CONTINUITY :

It is fundamental to finding the tangent to a curve or the velocity of an object. The geometric application of using limits to define the tangent to a curve leads at once to the important concept of the derivative of a function.

Let $f(x)$ be defined on an open interval about x_0, *except possibly at x_0 itself*. If $f(x)$ gets arbitrarily close to L (as close to L as we like) for all x sufficiently close to x_0, we say that f approaches the **limit** L as x approaches x_0, and we write

$$\lim_{x \to x_0} f(x) = L,$$

Limit Laws

If L, M, c and k are real numbers and

$$\lim_{x \to c} f(x) = L \qquad \text{and} \qquad \lim_{x \to c} g(x) = M, \quad \text{then}$$

1. *Sum Rule*: $\lim_{x \to c} (f(x) + g(x)) = L + M$

The limit of the sum of two functions is the sum of their limits.

2. *Difference Rule*: $\lim_{x \to c} (f(x) - g(x)) = L - M$

The limit of the difference of two functions is the difference of their limits.

3. *Product Rule*: $\lim_{x \to c} (f(x) \cdot g(x)) = L \cdot M$

The limit of a product of two functions is the product of their limits.

4. *Constant Multiple Rule:* $\lim_{x \to c} (k \cdot f(x)) = k \cdot L$

The limit of a constant times a function is the constant times the limit of the function.

5. *Quotient Rule*: $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{L}{M}, \quad M \neq 0$

The limit of a quotient of two functions is the quotient of their limits, provided the limit of the denominator is not zero.

6. *Power Rule*: If r and s are integers with no common factor and $s \neq 0$, then

$$\lim_{x \to c} (f(x))^{r/s} = L^{r/s}$$

provided that $L^{r/s}$ is a real number. (If s is even, we assume that $L > 0$.)

The limit of a rational power of a function is that power of the limit of the function, provided the latter is a real number.

EXAMPLE Using the Limit Laws

Use the observations $\lim_{x \to c} k = k$ and $\lim_{x \to c} x = c$ and the properties of limits to find the following limits.

(a) $\lim\limits_{x \to c} (x^3 + 4x^2 - 3)$ **(b)** $\lim\limits_{x \to c} \dfrac{x^4 + x^2 - 1}{x^2 + 5}$ **(c)** $\lim\limits_{x \to -2} \sqrt{4x^2 - 3}$

Solution

(a)
$$\begin{aligned} \lim_{x \to c} (x^3 + 4x^2 - 3) &= \lim_{x \to c} x^3 + \lim_{x \to c} 4x^2 - \lim_{x \to c} 3 && \text{Sum and Difference Rules} \\ &= c^3 + 4c^2 - 3 && \text{Product and Multiple Rules} \end{aligned}$$

(b)
$$\begin{aligned} \lim_{x \to c} \frac{x^4 + x^2 - 1}{x^2 + 5} &= \frac{\lim\limits_{x \to c} (x^4 + x^2 - 1)}{\lim\limits_{x \to c} (x^2 + 5)} && \text{Quotient Rule} \\ &= \frac{\lim\limits_{x \to c} x^4 + \lim\limits_{x \to c} x^2 - \lim\limits_{x \to c} 1}{\lim\limits_{x \to c} x^2 + \lim\limits_{x \to c} 5} && \text{Sum and Difference Rules} \\ &= \frac{c^4 + c^2 - 1}{c^2 + 5} && \text{Power or Product Rule} \end{aligned}$$

(c)
$$\begin{aligned} \lim_{x \to -2} \sqrt{4x^2 - 3} &= \sqrt{\lim_{x \to -2} (4x^2 - 3)} && \text{Power Rule with } r/s = \tfrac{1}{2} \\ &= \sqrt{\lim_{x \to -2} 4x^2 - \lim_{x \to -2} 3} && \text{Difference Rule} \\ &= \sqrt{4(-2)^2 - 3} && \text{Product and Multiple Rules} \\ &= \sqrt{16 - 3} \\ &= \sqrt{13} \end{aligned}$$

EXAMPLE Limit of a Rational Function

$$\lim_{x \to -1} \frac{x^3 + 4x^2 - 3}{x^2 + 5} = \frac{(-1)^3 + 4(-1)^2 - 3}{(-1)^2 + 5} = \frac{0}{6} = 0$$

This result is similar to the second limit in Example 1 with $c = -1$, now done in one step.

Evaluate

$$\lim_{x \to 0} \frac{\sqrt{x^2 + 100} - 10}{x^2}.$$

Solution This is the limit we considered in Example 10 of the preceding section. We cannot substitute $x = 0$, and the numerator and denominator have no obvious common factors. We can create a common factor by multiplying both numerator and denominator by the expression $\sqrt{x^2 + 100} + 10$ (obtained by changing the sign after the square root). The preliminary algebra rationalizes the numerator:

$$\begin{aligned}
\frac{\sqrt{x^2 + 100} - 10}{x^2} &= \frac{\sqrt{x^2 + 100} - 10}{x^2} \cdot \frac{\sqrt{x^2 + 100} + 10}{\sqrt{x^2 + 100} + 10} \\
&= \frac{x^2 + 100 - 100}{x^2\left(\sqrt{x^2 + 100} + 10\right)} \\
&= \frac{x^2}{x^2\left(\sqrt{x^2 + 100} + 10\right)} && \text{Common factor } x^2 \\
&= \frac{1}{\sqrt{x^2 + 100} + 10}. && \text{Cancel } x^2 \text{ for } x \neq 0
\end{aligned}$$

Therefore,

$$\begin{aligned}
\lim_{x \to 0} \frac{\sqrt{x^2 + 100} - 10}{x^2} &= \lim_{x \to 0} \frac{1}{\sqrt{x^2 + 100} + 10} \\
&= \frac{1}{\sqrt{0^2 + 100} + 10} && \text{Denominator not 0 at } x = 0\text{; substitute} \\
&= \frac{1}{20} = 0.05.
\end{aligned}$$

The Sandwich Theorem

Suppose that $g(x) \leq f(x) \leq h(x)$ for all x in some open interval containing c, except possibly at $x = c$ itself. Suppose also that

$$\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L.$$

Then $\lim_{x \to c} f(x) = L$.

EXAMPLE Applying the Sandwich Theorem

Given that

$$1 - \frac{x^2}{4} \leq u(x) \leq 1 + \frac{x^2}{2} \qquad \text{for all } x \neq 0,$$

find $\lim_{x \to 0} u(x)$, no matter how complicated u is.

Solution Since

$$\lim_{x \to 0} (1 - (x^2/4)) = 1 \qquad \text{and} \qquad \lim_{x \to 0} (1 + (x^2/2)) = 1,$$

the Sandwich Theorem implies that $\lim_{x \to 0} u(x) = 1$

Limit Laws as $x \to \pm\infty$

If L, M, and k, are real numbers and

$$\lim_{x\to\pm\infty} f(x) = L \quad \text{and} \quad \lim_{x\to\pm\infty} g(x) = M, \quad \text{then}$$

1. *Sum Rule:* $\lim_{x\to\pm\infty} (f(x) + g(x)) = L + M$
2. *Difference Rule:* $\lim_{x\to\pm\infty} (f(x) - g(x)) = L - M$
3. *Product Rule:* $\lim_{x\to\pm\infty} (f(x) \cdot g(x)) = L \cdot M$
4. *Constant Multiple Rule:* $\lim_{x\to\pm\infty} (k \cdot f(x)) = k \cdot L$
5. *Quotient Rule:* $\lim_{x\to\pm\infty} \dfrac{f(x)}{g(x)} = \dfrac{L}{M}, \quad M \neq 0$
6. *Power Rule:* If r and s are integers with no common factors, $s \neq 0$, then

$$\lim_{x\to\pm\infty} (f(x))^{r/s} = L^{r/s}$$

provided that $L^{r/s}$ is a real number. (If s is even, we assume that $L > 0$.)

EXAMPLE

(a) $$\lim_{x\to\infty}\left(5 + \frac{1}{x}\right) = \lim_{x\to\infty} 5 + \lim_{x\to\infty}\frac{1}{x} \qquad \text{Sum Rule}$$

$$= 5 + 0 = 5 \qquad \text{Known limits}$$

(b) $$\lim_{x\to-\infty}\frac{\pi\sqrt{3}}{x^2} = \lim_{x\to-\infty} \pi\sqrt{3}\cdot\frac{1}{x}\cdot\frac{1}{x}$$

$$= \lim_{x\to-\infty} \pi\sqrt{3}\cdot \lim_{x\to-\infty}\frac{1}{x}\cdot \lim_{x\to-\infty}\frac{1}{x} \qquad \text{Product rule}$$

$$= \pi\sqrt{3}\cdot 0 \cdot 0 = 0 \qquad \text{Known limits}$$